Comparisons and Combinations of Oscillation Measurements

May 11, 2004

1 Introduction

For a three active neutrino scenario, neutrino oscillations are described by six physics parameters: $\theta_{13}, \theta_{12}, \theta_{23}, \Delta m^2_{12}, \Delta m^2_{23}$, and the CP violation phase, δ. In addition, a full description also requires knowing the hierarchy of mass state 3 relative to 1 and 2, *i.e.* the sign of Δm^2_{23}. Of the six parameters, it is assumed for this study that $\theta_{12}, \theta_{23}, \Delta m^2_{12}$, and Δm^2_{23} are known to the precision expected from either the current program (SuperK, Minos and CNGS) or the future program (Nova and T2K). This leaves for determination θ_{13}, δ, and the mass hierarchy which are the subject of this study. Table 1 lists the values as well as the current and future errors used in the study for $\theta_{12}, \theta_{23}, \Delta m^2_{12}$, and Δm^2_{23}.

The experimental inputs for the study are given in Table 2 and are derived from estimates of the measurement sensitivities. Three reactor experiments are considered corresponding to a small (Double-CHOOZ), medium(Braidwood, Daya Bay type), or large (MiniBooNE size) reactor $\overline{\nu}_e$ measurement. Two offaxis long-baseline experiments are considered, JParc to SuperK (T2K) and the NuMI offaxis proposal (Nova). The sensitivities for the reactor experiments are scaled from the $\sin^2 2\theta_{13}$ 90% C.L. limits at $\Delta m^2_{23} = 2.5 \times 10^{-3}$ eV^2 for a null oscillation scenario. For the long-baseline experiments, the uncertainties are scaled from the expected number of events given in the Nova proposal and a recent talk by Y. Suzuki at the Seesaw workshop. The given uncertainties include statistical errors associated with the background and signal for a 5 year data run but no systematic uncertainty.

The uncertainty on the θ_{23} parameter can have a significant effect on the long-baseline measurements since the quantity that is constrained as given in Table 1 is $\sin^2 2\theta_{23}$ and the parameter that modulates the long-baseline oscillation probability is $\sin^2 \theta_{23}$. This can lead to a 65% (23%)

Parameter	Value	Current σ	Future σ
$\sin^2 2\theta_{23}$	1.0	0.06 (SuperK)	0.01 (T2K)
Δm^2_{23}(eV^2)	2.5×10^{-3}	0.33×10^{-3} (SuperK)	0.05×10^{-3} (T2K)
θ_{12}(deg)	30	–	–
Δm^2_{12}(eV^2)	7.1×10^{-5}	–	–

Table 1: Current and future uncertainty estimates on oscillation parameters. This study assumes values corresponding to the future estimates.

	Basis of	Osc. Prob. and σ for $\sin^2 2\theta_{13} =$		
Experiment	Estimate	0.02	0.05	0.10
Reactor ($E_\nu = 3.6$ MeV)	$\sin^2 2\theta_{13}^{Limit}$			
$\langle L \rangle$	@ $\Delta m^2_{23} = 2.5 \times 10^{-3}\mathrm{eV}^2$			
Small 1.05 km	$0.03@90\%CL$	0.013 ± 0.018	0.032 ± 0.018	0.064 ± 0.018
Medium 1.8 km	$0.01@90\%CL$	0.020 ± 0.006	0.050 ± 0.006	0.100 ± 0.006
Large 1.8 km	$0.005@90\%CL$	0.020 ± 0.003	0.050 ± 0.003	0.100 ± 0.003
T2K ($E_\nu = 600$ MeV)	$\mathrm{N}^{5yrs}_{events}$: $\sin^2 2\theta_{13} = 0.1$, $\delta_{CP} = 0$			
$\langle L \rangle = 295$ km	@ $\Delta m^2_{23} = 2.5 \times 10^{-3}\mathrm{eV}^2$			
ν	102 signal / 24.9 bkgnd	0.011 ± 0.003	0.026 ± 0.004	0.051 ± 0.005
$\overline{\nu}$	38.5 signal / 14.4 bkgnd	0.009 ± 0.006	0.022 ± 0.007	0.044 ± 0.009
Nova ($E_\nu = 2.3$ GeV)	$\mathrm{N}^{5yrs}_{events}$: $\sin^2 2\theta_{13} = 0.1$, $\delta_{CP} = 0$			
$\langle L \rangle = 810$ km	@ $\Delta m^2_{23} = 2.5 \times 10^{-3}\mathrm{eV}^2$			
ν	175.2 signal / 38.1 bkgnd	0.011 ± 0.002	0.025 ± 0.003	0.048 ± 0.003
$\overline{\nu}$	66 signal / 22 bkgnd	0.008 ± 0.003	0.018 ± 0.004	0.034 ± 0.005

Table 2: Estimates of the experimental uncertainties associated with various future oscillation experiments. For the long-baseline experiments, the given uncertainties include statistical errors associated with the background and signal for a 5 year data run but no systematic uncertainty.

uncertainty in the oscillation probability with the present (future) errors.

For the studies given below, the uncertainties due to the variations of θ_{23}, Δm^2_{23}, and the mass hierarchy are included. For $\overline{\nu}$ running, there is can be up to a 20% contamination of ν oscillation events; for these studies, the contamination is assumed to be zero. Results are typically given for five year data runs but in addition some results are presented for ×5 the nominal rate (or 25 effective years) which would somewhat correspond to an upgraded long-baseline program with a new proton driver at Fermilab or the Hyper-K upgrade at JParc.

2 Determination of θ_{13}

The leading order dependence of the oscillation probability for reactor and long-baseline measurements is given by

$$\begin{aligned} P_{reactor} &= 1 - P(\nu_e \to \nu_e) \simeq \sin^2 2\theta_{13} \sin^2\left(1.27 \Delta m^2_{31} L/E\right) \\ P_{long-baseline} &= P(\nu_\mu \to \nu_e) \simeq \sin^2 2\theta_{13} \sin^2 \theta_{23} \sin^2\left(1.27 \Delta m^2_{31} L/E\right) \\ &\quad + \text{ Other terms with CP viol. and matter effects} \end{aligned}$$

In the investigations shown here, the full formulae for the oscillation probability have been used as incorporated in a computer program developed by Stephen Parke. The higher order corrections for the reactor probability are quite small and a measurement of $P_{reactor}$ is simply related to a constraint on the mixing parameter θ_{13}. On the other hand, the full expression for the long-baseline probability introduces many degeneracies and correlations between the physics parameters θ_{23} and

δ_{CP} plus the mass hierarchy through matter effects. Therefore, a measurement of $P_{long-baseline}$ corresponds to regions in the physics parameter space.

As an indication of how well a given measurement can constrain the value of θ_{13}, Figure 1 (top: T2K, bottom: Nova) shows the 90% C.L. allowed regions associated with measurements of a null oscillation scenario where $\sin^2 2\theta_{13} = 0$. The three vertical dashed lines correspond to the 90% C.L. upper limits from a large, medium, and small reactor measurement as presented in Table 2. The green region (white curve) is the 90% C.L. allowed region for the two long-baseline experiments for a five year neutrino only run with the nominal ($\times 5$) beam rate. Combining the long-baseline and medium reactor measurement gives the improved blue region. In all cases, the results include the variations associated with the uncertainty in θ_{23}, Δm^2_{23}, and mass hierarchy.

If a θ_{13} is large enough, then positive signals will be observed by the experiments. Under these circumstances, the goal would be to make the best determination of the mixing parameter. Figure 2 shows the 90% C.L. regions that will be obtained for an underlying scenario where $\sin^2 2\theta_{13} = 0.05$. It is clear that a long-baseline only measurement will not determine the mixing parameter very well with an allowed region that spans from 0.02 to over 0.11. On the other hand, a reactor experiment with at least the medium scale sensitivity measures $\sin^2 2\theta_{13}$ to about 10% and θ_{13} to $\pm 0.4^\circ$.

As seen from the figures, the reactor measurements are very efficient at constraining the value of θ_{13} and even a small reactor experiment can probe for an early indication that the value is sizeable. The large reactor experiment has sensitivity comparable to planned long-baseline experiments and the medium scale experiment can measure values in the range above $\sin^2 2\theta_{13} > 0.02$ at the 10% to 20% level. As will be seen in later plots, studies of CP violation and matter effects over the next decade are only possible if $\sin^2 2\theta_{13}$ is significantly larger than about 0.01. A medium scale reactor experiment can set the scale if these studies will be possible and, if they are, add additional information to the determination of the parameters.

3 Constraining CP Violation Parameters

One of the important goals of an oscillation physics program is to determine if CP violating effects are present in the lepton sector as probed through the neutrino mixing matrix. In contrast to the reactor disappearance probability, the oscillation probabilities for the long-baseline experiments are effected by the value of the CP violation phase δ_{CP}. Due to these different types of behavior, combinations of long-baseline neutrino, anti-neutrino, and reactor measurements can be used to isolate these CP violating effects and place constraints on δ_{CP}. The size of these effects are scaled by the value of $\sin^2 2\theta_{13}$ which is therefore an important parameter for setting the sensitivity to CP violation. The analysis also needs to include the uncertainties associated with the other parameters and especially the mass hierarchy. Figure 3 gives the $\nu_\mu \rightarrow \nu_e$ appearance oscillation probability as a function of δ_{CP} for the various combinations of beam type and mass hierarchy for $\sin^2 2\theta_{13} = 0.06$.

From these figures, it can be seen that a measurement of the appearance probability for neutrino running alone could give information on δ_{CP} if the value of $\sin^2 2\theta_{13}$ was known for example from a reactor oscillation measurement. Figure 4 shows that the combinations of various measurements can be used to constrain the allowed CP violation phase. The results are for a scenario with $\sin^2 2\theta_{13} = 0.05$ and the optimum phase point $\delta_{CP} = 270^\circ$. In the top plot, a T2K ν–only (5 years)

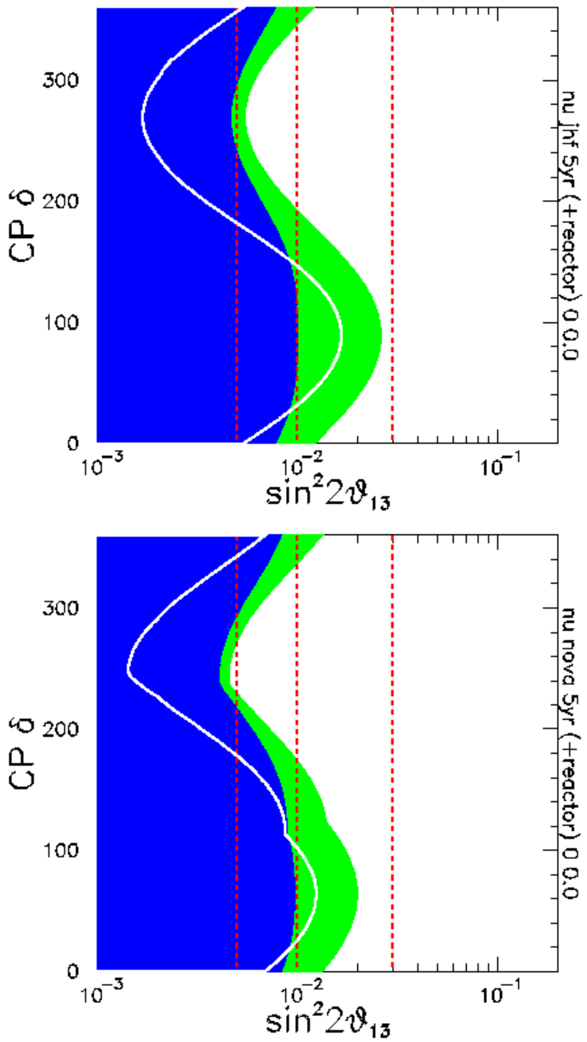


Figure 1: 90% C.L. regions and upper limits for various oscillation measurements for an underlying null oscillation scenrio where $\sin^2 2\theta_{13} = 0$. .The The top (bottom) plot is for the T2K (Nova) long-baseline experiments. The three vertical dashed lines correspond to the 90% C.L. upper limits from a large, medium, and small reactor measurement as presented in Table 2. The green region (white curve) is the 90% C.L. allowed region for the two long-baseline experiments for a five year neutrino only run with nominal (×5) beam rate and the blue region gives the combination of the five year long-baseline and medium reactor measurements.

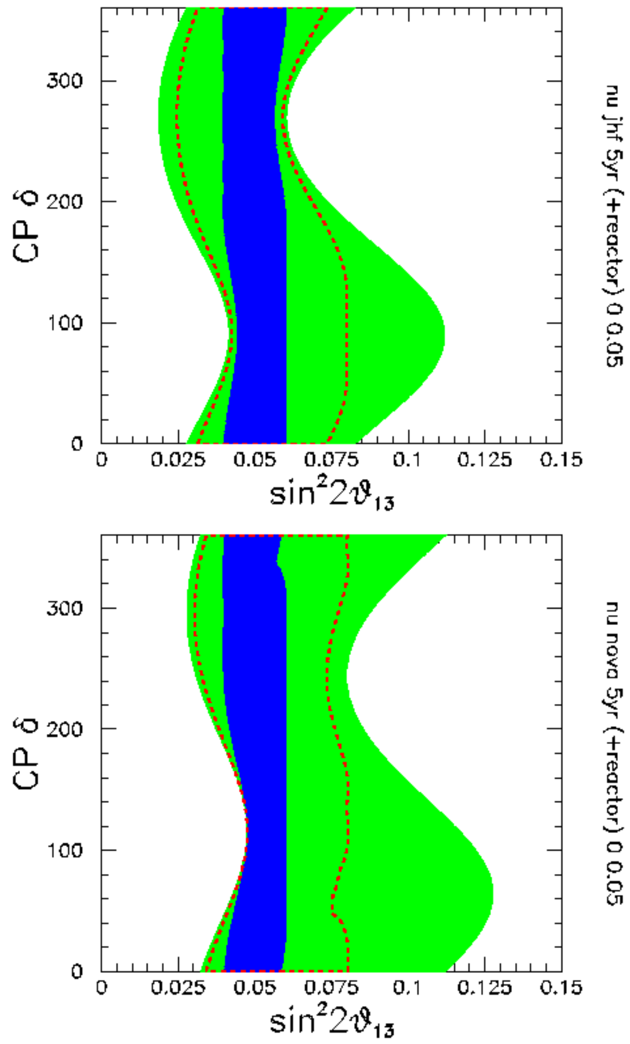
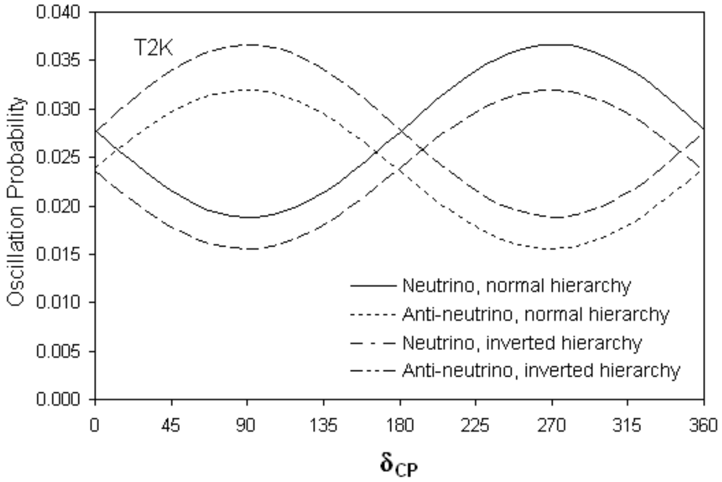


Figure 2: 90% C.L. regions for underlying oscillation parameters of $\sin^2 2\theta_{13} = 0.05$, $\Delta m^2 = 2.5 \times 10^{-3}$ eV^2 and $\delta_{CP} = 0$. The green regions are for the T2K (top plot) or Nova (bottom plot) experiments for five years of neutrino running. The blue regions are the 90% C.L. allowed regions for a combined medium reactor plus long-baseline analysis. The dashed red lines indicate how the combined measurement would degrade with the small reactor sensitivity.



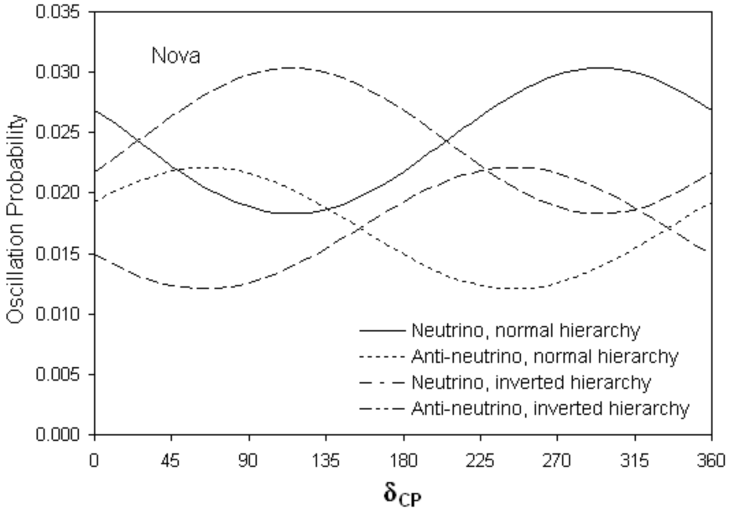


Figure 3: Oscillation probability for $\nu_\mu \rightarrow \nu_e$ appearance vs. δ_{CP} for the T2K (top) and Nova (bottom) experimental setups with $\Delta m^2 = 2.5 \times 10^{-3}$ and $\sin^2 2\theta_{13} = 0.05$. The four curves correspond to pure neutrino or anti-neutrino beams with a normal or inverted hierarchy.

measurement is displayed first without any reactor measurement (green region) then combined with a medium scale reactor measurement (blue region). (The dashed red curve outlines the region using a small scale reactor result.) The middle then shows what happens when both neutrinos (5 years) and anti-neutrinos (5 years) are used with and without the reactor measurement. Finally, the bottom plot shows the combination of T2K and Nova with and without the reactor result. The reactor measurement allows an early investigation of CP violation with a ν–only long-baseline measurement and in all cases significantly reduces the uncertainty on θ_{13}.

As a measure of how well the CP phase can be constrained in general, Figure 5 gives the regions in the $\delta_{CP} - \sin^2 2\theta_{13}$ plane for which a null CP violation solution is ruled out by at least two standard deviations. The dashed curves use long-baseline data only and the solid curves add data from a medium scale reactor experiment. The upper panels are for 5 years of running with neutrinos and antineutrinos assuming the rates shown in Table 2. Plot a) is for Nova only and b) combines data from both T2K and Nova. For the lower panels, it is assumed that there is a $\times 5$ increase in intensity due, for example, to adding a Proton Driver at Fermilab. Plot c) is for Nova and d) combines Nova and T2K. From the plots, it is seen that the combination of T2K and Nova can probe the CP violation phase space if $\sin^2 2\theta_{13} \gtrsim 0.05$ (0.02) for nominal ($\times 5$) beam rates. One also sees that the reactor experiment is important for resolving the degeneracy at $\delta_{CP} = 180°$.

4 Determining the Mass Hierarchy

For constraining the mass hierarchy, one needs to compare measurements in a region where the oscillation probability changes significantly for a normal versus inverted mass spectrum (See Figure 3). The Nova experiment is particularly important here due to their long pathlength in matter. Figure 6 shows the Nova $\nu_\mu \to \nu_e$ oscillation probability versus $\sin^2 2\theta_{13}$. The two sets of vertical bars show the variation with δ_{CP} for the normal (top) and inverted (bottom) hierarchy. A medium scale reactor experiment will constrain the value of the $\sin^2 2\theta_{13}$ to the region delineated by the vertical lines. Combining this with a 5 year Nova measurement of the oscillation probability could, if the parameters are favorable, determine which hierarchy was allowed.

A more accurate determination of the hierarchy is possible by combining the results from long-baseline neutrino and antineutrino data. Here again the ambiguity with respect to the value of δ_{CP} limits the determination to regions in the $\sin^2 2\theta_{13} - \delta_{CP}$ plane. Figure 7 shows the regions for which the mass hierarchy is resolved by two standard deviations. The dashed curves use long-baseline data only and the solid curves add data from a medium (or large) scale reactor experiment. The various panels in the figure are for data sets given by: a) Nova for ν(5yr) only data and ν(3yr) plus $\overline{\nu}$(3yr) data; b) Nova plus T2K for ν(3yr) plus $\overline{\nu}$(3yr) data; c) Nova with $\times 5$ the nominal beam rate for ν(3yr) plus $\overline{\nu}$(3yr) data; d) T2K and Nova with $\times 5$ the nominal beam rate, each for ν(3yr) plus $\overline{\nu}$(3yr).

The plots show that the mass hierarchy can be determined for limited regions with $\sin^2 \theta_{13} > 0.05$ for the nominal beam rates and > 0.025 for the enhanced ($\times 5$) rates. The addition of the T2K and reactor data helps improve the measurement reach for the bad δ_{CP} regions.

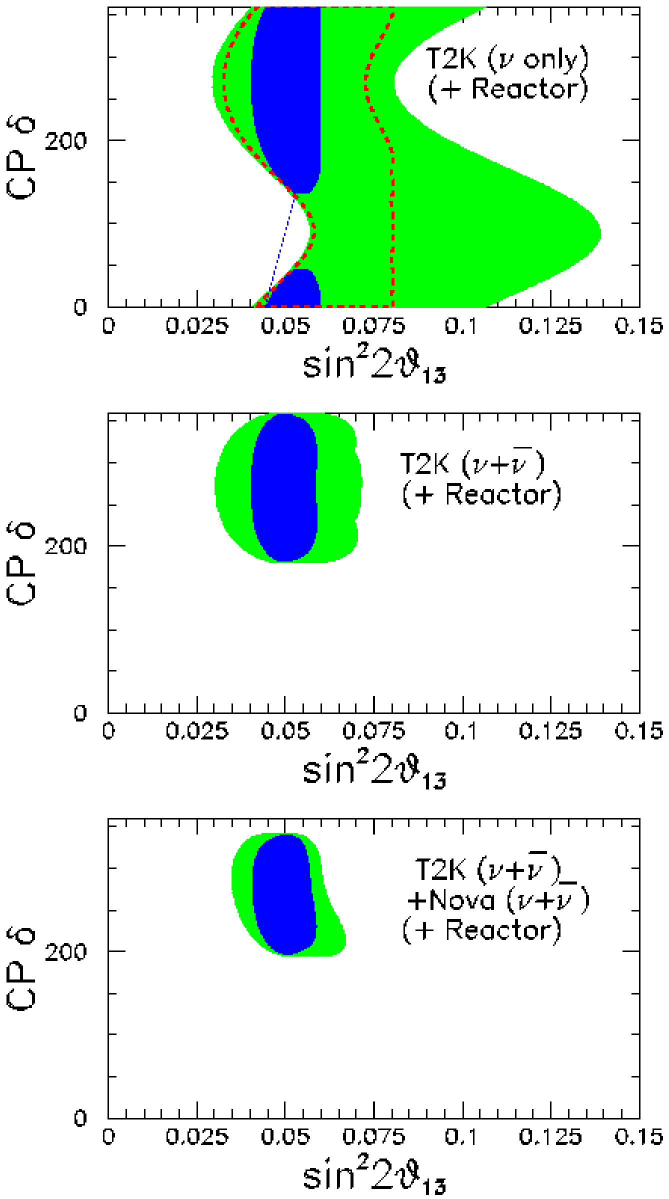


Figure 4: 90% C.L. regions for various combinations of oscillation results for $\sin^2 2\theta_{13} = 0.05$ and $\delta_{CP} = 270°$. Top: 5 year ν–only T2K data with/without medium scale reactor results. Middle: T2K $\nu + \overline{\nu}$ with/without reactor result. Bottom: T2K $\nu + \overline{\nu}$ plus Nova $\nu + \overline{\nu}$ with/without reactor result.

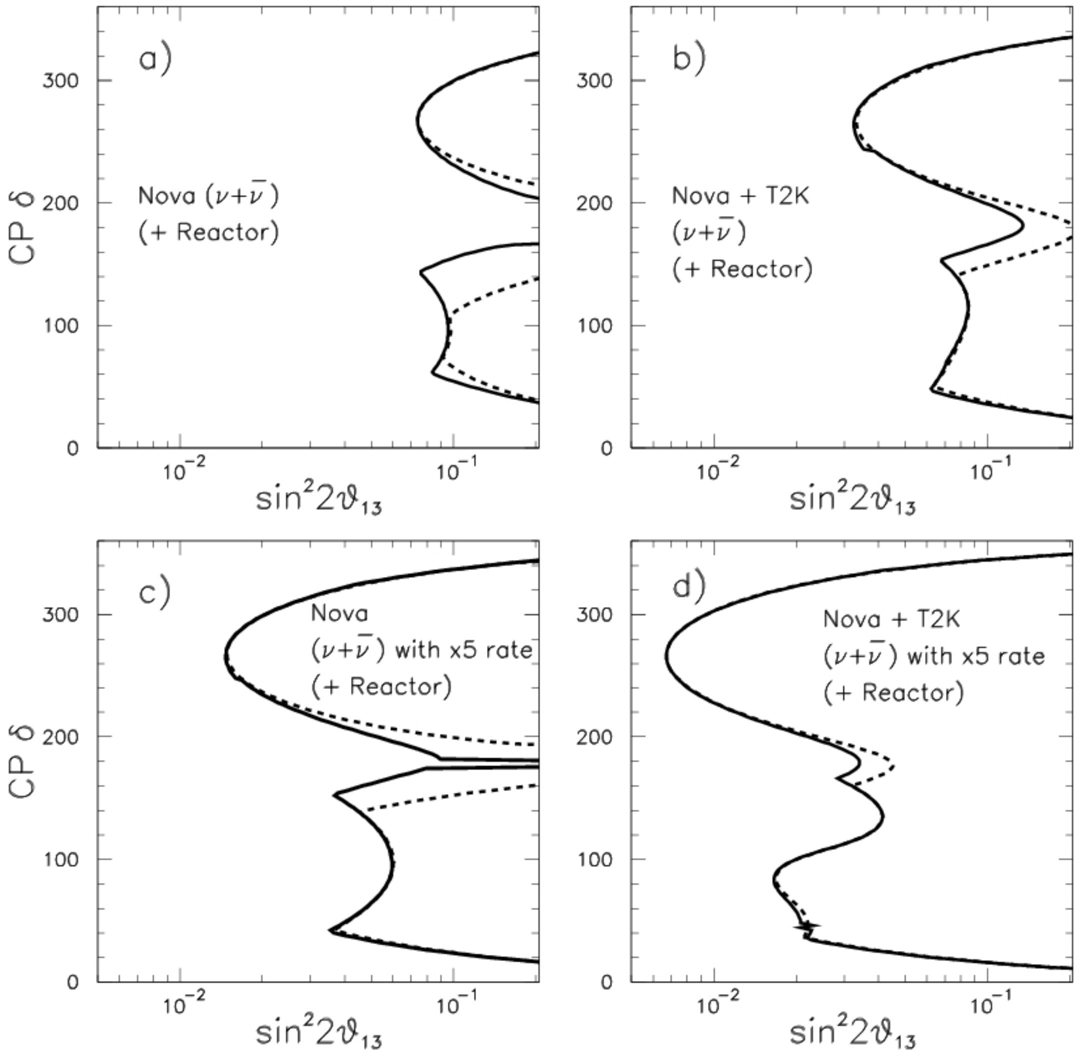


Figure 5: Regions in the $\delta_{CP} - \sin^2 2\theta_{13}$ plane for which a null CP violation solution is ruled out by at least two standard deviations. The dashed curves use long-baseline data only and the solid curves add data from a medium scale reactor experiment. a) Nova ν(5yr) + $\overline{\nu}$(5yr) data; b) T2K ν(5yr) + $\overline{\nu}$(5yr) + Nova ν(5yr) + $\overline{\nu}$(5yr) data; c) Nova (×5 rate with Proton Driver) ν(5yr) + $\overline{\nu}$(5yr) data; d) T2K (×5 rate) ν(5yr) + $\overline{\nu}$(5yr) + Nova (×5 rate) ν(5yr) + $\overline{\nu}$(5yr) data

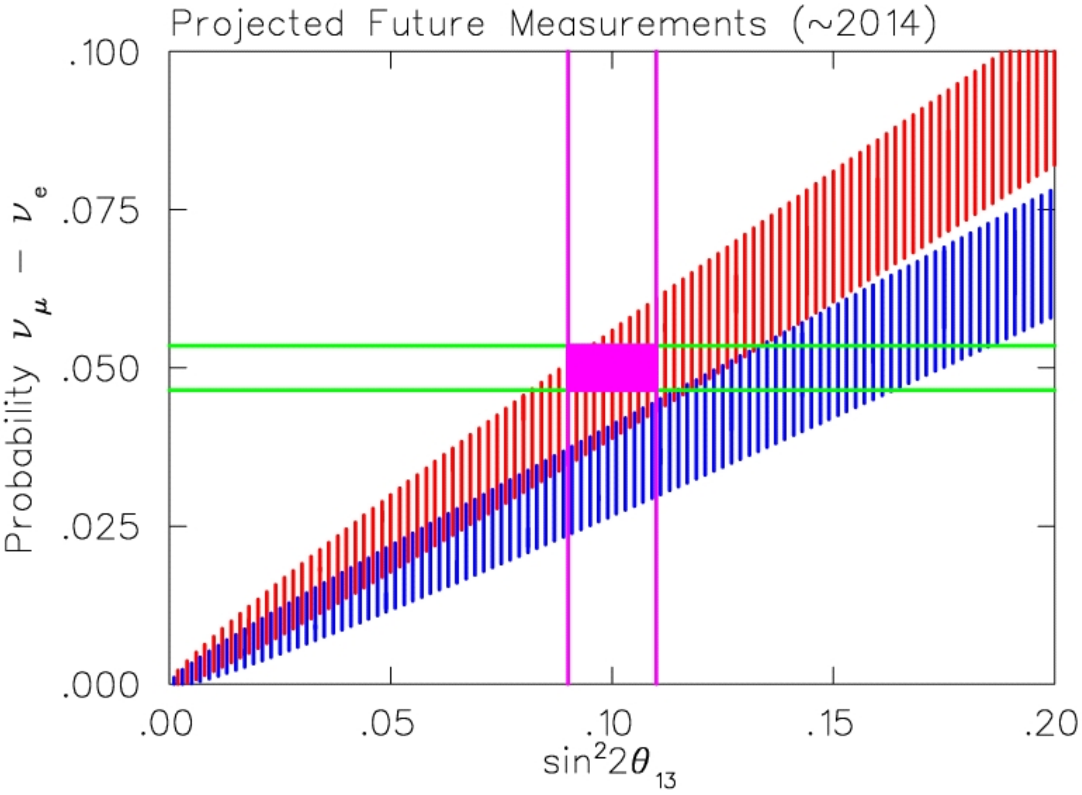


Figure 6: Example of how a medium scale reactor measurement and a 5 year ν–only Nova measurement can constrain the mass hierarchy.

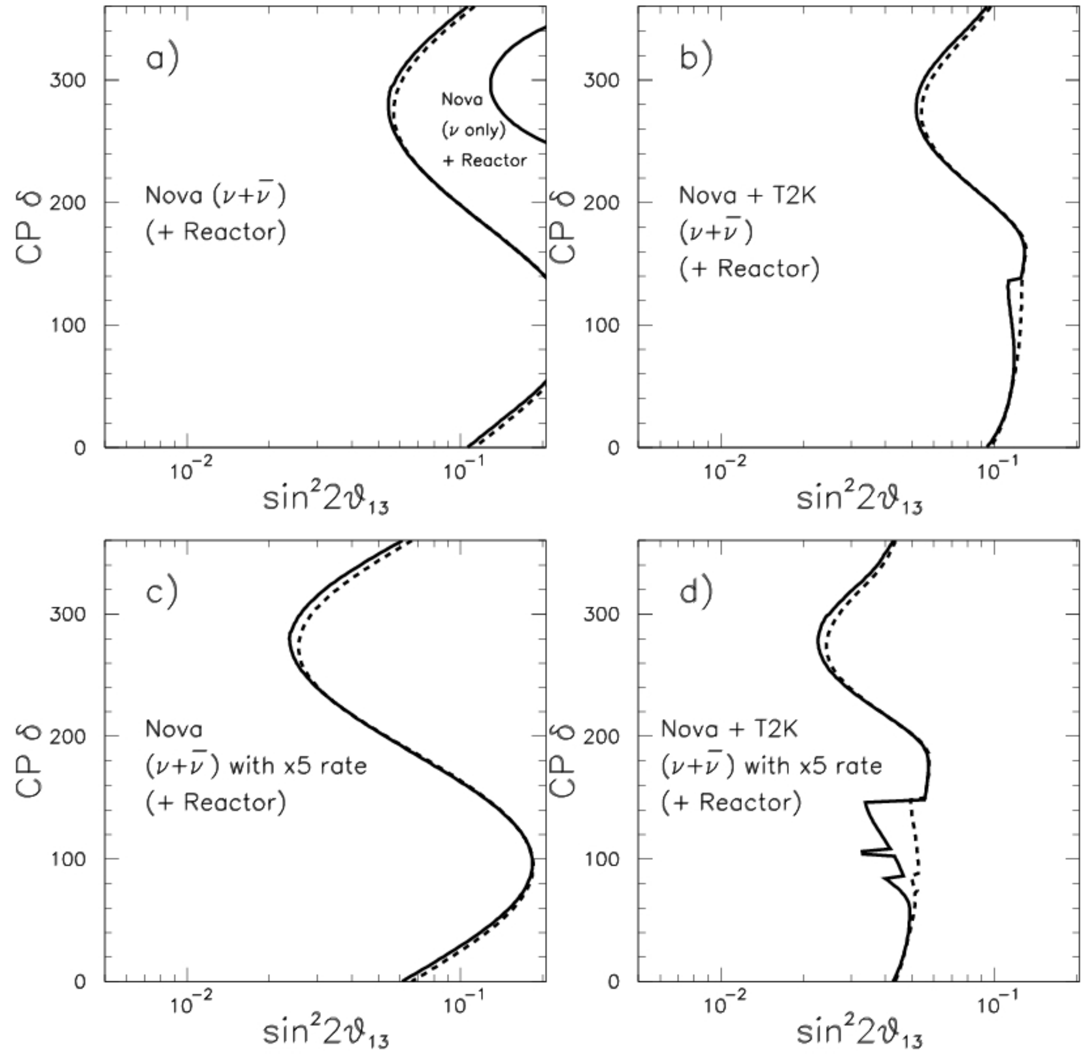


Figure 7: Regions in the $\delta_{CP} - \sin^2 2\theta_{13}$ plane for which the mass hierarchy is resolved by two standard deviations. The dashed curves use long-baseline data only and the solid curves add data from a medium scale reactor experiment. a) Nova ν(5yr) only data and ν(3yr) + $\bar{\nu}$(3yr) data; b) Nova plus T2K with ν(3yr) + $\bar{\nu}$(3yr) data; c) Nova (×5 beam rate) with ν(3yr) + $\bar{\nu}$(3yr) data; d) T2K (×5 beam rate rate) ν(3yr) + $\bar{\nu}$(3yr) + Nova (×5 beam rate) ν(3yr) + $\bar{\nu}$(3yr) data.

5 Conclusions

The worldwide program to understand neutrino oscillations and determine the mixing parameters, CP violating effects, and mass hierarchy will require a broad combination of measurements. Progress in the past associated with solving the solar and atmospheric neutrino puzzles took a full suite of experiments to isolate and understand the phenomenology. As measurements became available, they defined the direction for future studies. One would expect a similar chain for the current goals where the program grows as information is obtained.

Reactor measurements hold the promise of constraining or measuring the θ_{13} mixing parameter. In addition to setting the scale for further studies, a reactor result when combined with long-baseline measurements may also give early indications of CP violation and the mass hierarchy. The combination of the T2K and Nova longbaseline experiments will be able to make significant measurements of these effects if $\sin^2 2\theta_{13} > 0.02$ and with enhanced beam rates can improve their reach to the $\sin^2 2\theta_{13} > 0.02$ level. If theta13 turns out to be smaller than these values, one will need other stategies for getting to the physics. Thus, an unambiguous reactor measurement of θ_{13} is an important ingredient in planning the strategy for this program.